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The Blending Efficiencies of Some Impellers in Batch Mixing

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Mixing times for the blending of viscous liquids with tracer materials have been measured in a mechanically agitated tank using two conventional agitator types (turbine, helical ribbon) and a series of novel tubular-type agitators. The range of conditions studied were: viscosity = 1 to 70,000 cP, agitator Reynolds number = 5 to 10^5, tank diameter = 11.5 in. It was found that in terms of agitator power requirement, the tubular agitators were generally more efficient for viscosities greater than 100cP than the others for the blending process.

The effects of pseudoplastic non-Newtonian fluid behavior, of tank-baffles, and of tank size on mixing times have also been examined in several of the above cases.

"What is most needed in agitation research at the present time is not extensive correlation of power consumption of inefficient mixing devices, but intensive study directed toward design and selection of impellers which will utilize the power invested more efficiently" (*1*). "For economy of agitation it is, however, important to ensure that the impeller operates for the shortest time under such conditions which permit the required agitation. When judging the power consumption of an impeller it is then necessary to take into account the total energy consumption during the time necessary for the accomplishment of the required mixing effect" (*2*).

Relatively little work has been done on the comparative evaluation of various impellers with regard to the efficiency with which they utilize power. Within this context, an attempt has been made to compare a few conventional impeller types with some novel nonconventional ones. Specifically, the relationship between mixing time and power consumption (as the two basic parameters characterizing mixing efficiency of the impellers) was experimentally examined. The experiments were limited to the blending of miscible liquids of the same viscosity and density.

EXPERIMENT

Most of the experiments were conducted in a cylindrical vessel, 11½ in. in diameter, with standard or almost standard geometric configurations (*2*). For scaling up inferences, a few runs were also performed in a 16-in. diameter vessel. The liquid height was fixed at one tank diameter in all experiments, and the impellers were operated in the usual central position. For baffled conditions, the common arrangement of four wall baffles each of width equal to 1/10 the tank diameter, were used.

The following impeller types were tested (for additional details, see Figure 1 and Notation).

1. 6-bladed flat turbine: D = 4 in., C = 1/3T, w = 1 in., h = 0.75 in.
2. helical ribbon with single flight: D = 8 in., w = 0.75 in., h = 11.5 in., pitch = 4 in.
3. single tubular agitator I: D = 6.25 in., I.D. of main tube = 1.03 in. (O.D. = 1.2 in.), I.D. of arms = 0.63 in. (O.D. = 0.75 in.), C = 2.5 in., C′ = 8.5 in.
4. single tubular agitator II: D = 7.0 in., I.D. of main tube = 0.69 in., (O.D. = 0.75 in.), I.D. of arms = 0.44 in., (O.D. = 0.50 in.), C = 1.5 in., C′ = 8 in.
5. double tubular agitator I: D (upper) = 7.0 in., D (lower) = 4.5 in., I.D. of main tube = 0.69 in. (O.D. = 0.75 in.), I.D. of arms (upper) = 0.44 in. (O.D. = 0.50 in.), I.D. of arms (lower) = 0.31 in. (O.D. = 0.375 in.), C = 1.5 in., C_1 = 8 in., C_2 = 4 in.
6. double tubular agitator II: D (upper and lower) = 7.0 in., I.D. of main tube = 0.69 in. (O.D. = 0.75 in.), I.D. of arms (upper and lower) = 0.44 in. (O.D. = 0.50 in.), C = 1.5 in., C_1 = 8 in., C_2 = 4 in.

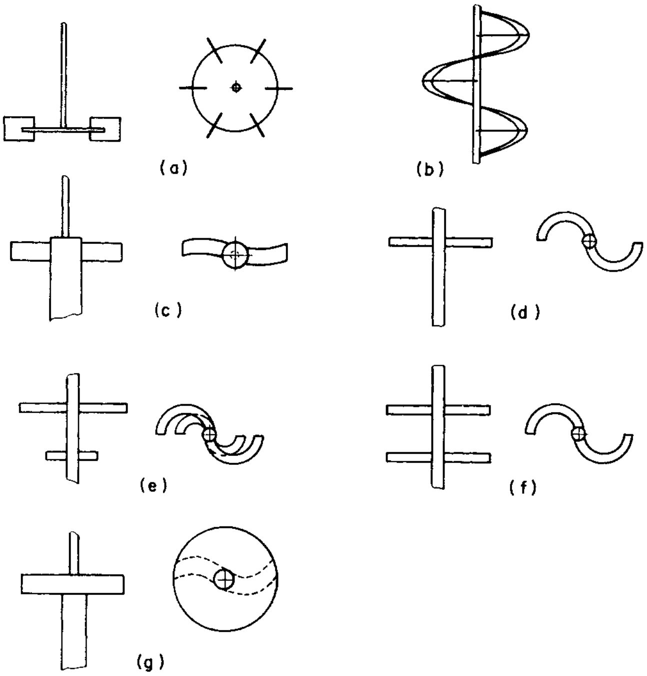

Fig. 1. Configuration of agitators used.

7. single tubular agitator with disc: D = 4 in., C = 1.5 in., C′ = 10.5 in., thickness of disc = 1 in.

The tubular agitators represented modifications of certain unconventional patented impellers (*3, 4*). The principle of their action is based on the ejection of liquid contained inside the horizontal tubes due to centrifugal action and subsequent refilling of the tubes from the bulk liquid in the tank. One of these devices (*3*) is actually used in the pulp and paper industry. The major difference between the tubular agitators tested in the present work and those patented was that in the present case the liquid was drawn in at the bottom of the tank and ejected higher up, about 2/3 of the way from the bottom of the tank. It was hoped that axial mixing of the liquid would be improved with this arrangement.

The Newtonian liquids used were water and aqueous solutions of commercial sugar and glucose 43 (St. Lawrence Starch Company) having viscosities ranging from 1 to 70,000 cP. The non-Newtonian liquids used were aqueous solutions of sodium carboxymethyl cellulose, a commercially available thickening agent (Hercules Company).

Mixing times were measured using a common decolorization method previously described by other workers (*1, 2*). An acid-base reaction between sodium hydroxide and hydrochloric acid with a mixture of three parts of 0.1% bromocresol green in alcohol and one part 0.2% methyl red in alcohol as indicator was used. The color change is from green in an alkaline solution to red in an acidic one. For each test 15 ml. of 1N solutions of tracer (hydrochloric acid or sodium hydroxide as the case may be) was prepared from the bulk liquid at the same viscosity and density and was injected into the vessel by means of an hypodermic syringe at a height 1/3 from the bottom of the vessel. The mixing time t_p in the study was taken as the time in which p% of the vessel volume was completely mixed (indicated by complete color change). These measurements were reproducible to within ±15%.

An extremely sensitive method of measuring impeller torque employing air-bearings previously developed by Calderbank and Moo-Young (*5, 6*) was used. The error in the measured torque was within ±1%. The rev./min. was accurately determined by using a perforated disc attached to the shaft of the motor. While rotating, the disc caused periodic changes in illumination of a photocell via optical fibers. The output of the photocell was supplied to an electronic rev./min. counter.

For viscometric measurements a Couette-type Brookfield Synchro-Lectic viscometer was used. Rheological properties of the non-Newtonian liquids obeyed the usual power law:

$$\tau = K \dot{\gamma}^n \tag{1}$$

over the shear rate range of 1 to 100 sec.$^{-1}$, using the equation derived by Calderbank and Moo-Young (*6*)

$$\dot{\gamma}_{\text{non-Newtonian}} = C_R \; \dot{\gamma}_{\text{Newtonian}} \tag{2}$$

where

$$C_R = 1 + \frac{S^2 - 1}{2S^2}\left(\frac{1}{m} - 1\right)\left(1 + \frac{2}{3}\ln S + \frac{1}{3}\dot{\gamma}^3 - \frac{1}{45}\dot{\gamma}^3 + \frac{2}{945}\dot{\gamma}^5 - \dots\right)$$

in which

$$\dot{\gamma} = \left(\frac{1}{m} - 1\right)\ln S$$

and m is the slope of a logarithmic plot of shear stress versus rotational speed. Fluid density measurements were carried out with an immersion hydrometer.

RESULTS AND DISCUSSION

Correlation of Data

It has been pointed out that "with certain limitations we are now able to make a reliable estimate of the power required to turn the impeller of a standard design at any speed in any environment" (*1*), and the following form of a power correlation is commonly used

$$Po = ARe^a \tag{3}$$

where

$$Po = \frac{Pg_c}{\rho N^3 D^5}$$

$$Re = \frac{D^2 N^{2-n} \rho B^{1-n}}{K}\left(\frac{4n}{3n+1}\right)^n$$

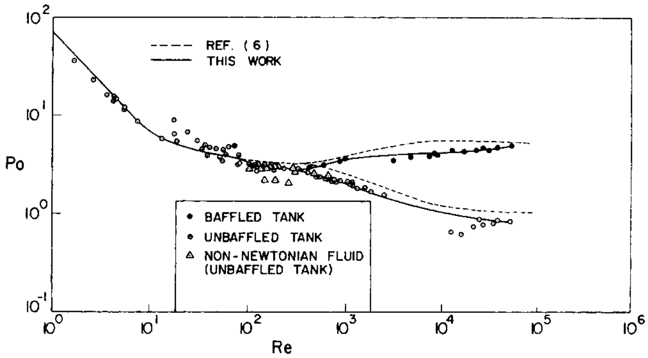

Fig. 2. Correlation of agitation power consumption (turbine).

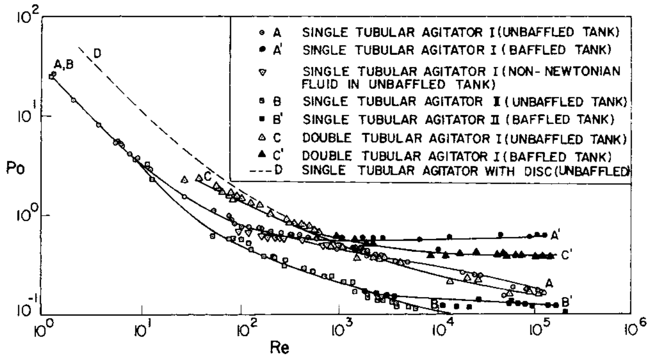

Fig. 3. Correlation of agitation power consumption (tubular agitators). Data points for *D* omitted for clarity.

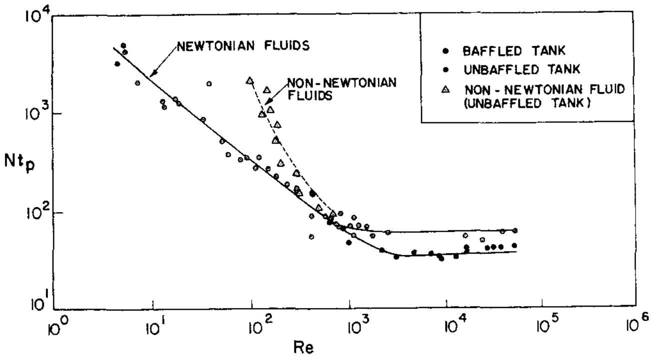

Fig. 4. Correlation of mixing time (turbine).

TABLE 1. SUMMARY OF MIXING TIME PARAMETERS FOR MAJOR AGITATORS ACCORDING TO EQUATION (4)

Agitator type	*Range of Re*	*A′*	*−a′*
Turbine	$5 \times 10^0 - 10^3$	1.75×10^4	0.75
	$10^3 - 10^5$	60	0
Turbine (baffled)	$10^3 - 10^5$	36	0
Helical ribbon	$10^0 - 10^4$	4.3×10^2	0.25
Tubular agitator (with disc)	$50 - 2 \times 10^2$	1.6×10^6	2.25
	$2 \times 10^2 - 3 \times 10^3$	4.5	0.11
Single tubular agitator I	$10 - 5 \times 10^2$	2.2×10^4	0.85
	$10^3 - 10^5$	44	0
Single tubular agitator II	$10 - 5 \times 10^2$	4.2×10^4	0.85
	$10^3 - 10^5$	56	0

and, A, B, and a are empirical constants for a given equipment geometry (6).

In Figure 2 the data obtained for the turbine impeller have been plotted in the form of the above standard Power number-Reynolds number correlation and compared with those previously obtained by Calderbank and Moo-Young (6). It is seen that the two sets of data are in fairly good agreement. A similar plot (not shown) have also been obtained for the helical ribbon in agreement with previous findings (7). Figure 3 shows that Equation (3) is also applicable to the tubular agitators. In all these plots the laminar-flow regime is characterized by a negative slope of unity as found by previous workers for the conventional agitators.

Although there are several types of mixing time correlations the present results were examined in terms of the following which is probably the one most commonly used (*1*, *8*):

$$Nt_p = A'Re^{a'} \tag{4}$$

where A' and a' are empirical constants for a given geometry. The form of Equation (4) was indeed applicable to the conventional agitators (for example, Figure 4) as well as to the tubular agitators (Figure 5).

Inspection of the various figures shows a similarity between the power and mixing time correlations and in the region of laminar flow, both relations are linear. However, while in the power number correlations the slope of the line is the same for all impellers, this is not true for the mixing time correlations where the slope of the line is different for the various impellers used. Table 1 summarizes these variations in the mixing time parameters for the agitators of primary interest.

Mixing time as a function of power consumption

Correlations of mixing time and power consumption outlined in the previous section make it possible to express mixing time in terms of power consumption for a given impeller in a given geometric configuration for arbitrary hydrodynamic conditions as defined by the Reynolds number.

One direct way of comparing various impellers from the point of view of their mixing efficiency is to relate the power consumption with mixing time for various viscosities. Typical results for a viscosity of 1,480 cP and of 1 cP are shown in Figures 6 and 7. The product of the power and the corresponding mixing time gives the amount of energy used to achieve the indicated degree of mixing. The general conclusion from a series of these plots is that the various tubular impellers and also the helical ribbon as found by previous workers perform much better than the turbine for nonturbulent flow conditions whereas for mixing of the liquids in the turbulent flow regimes no sig-

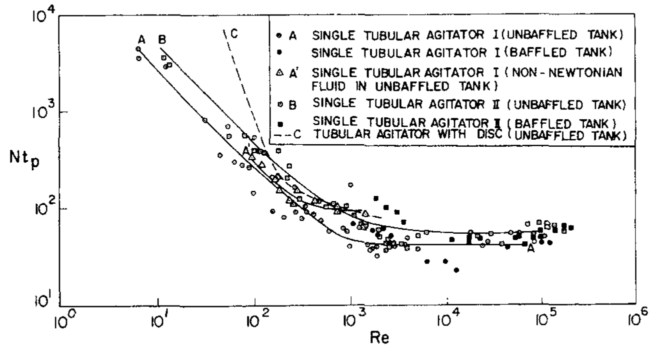

Fig. 5. Correlation of mixing time (tubular agitators). Data points for C omitted for clarity.

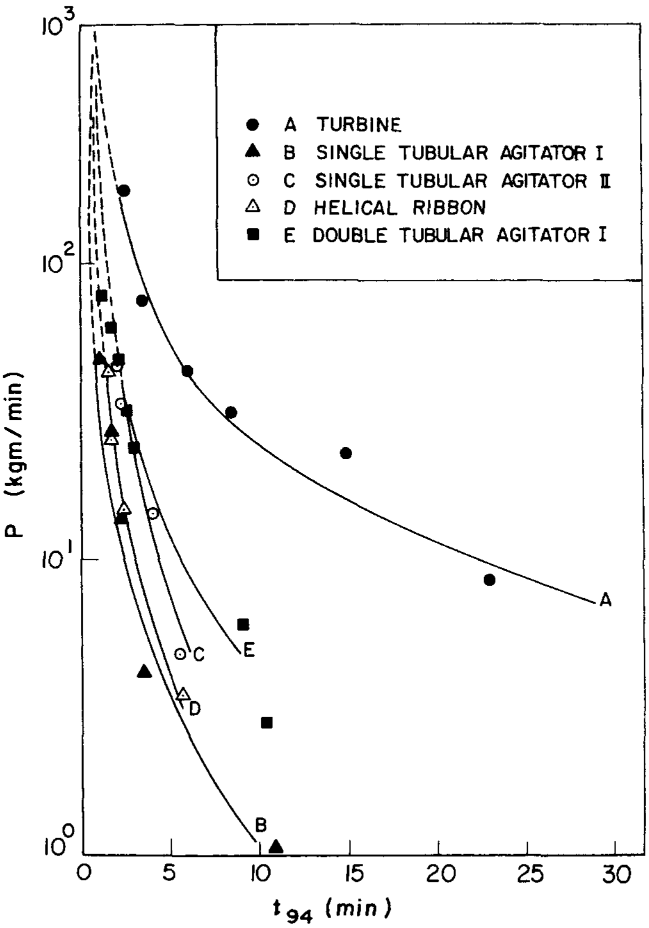

Fig. 6. Mixing time as a function of power consumption (unbaffled tank) $\mu = 1480$ cP.

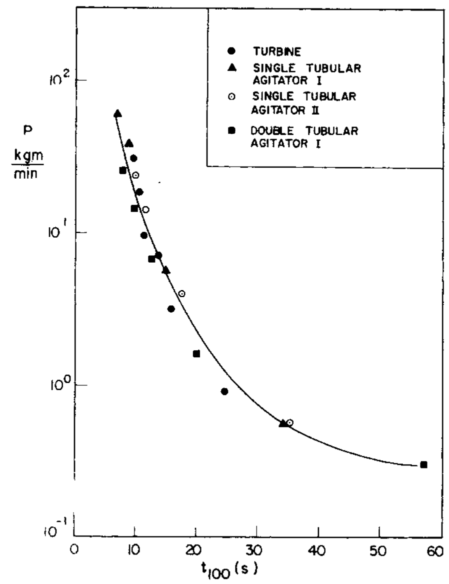

Fig. 7. Mixing time as a function of power consumption (unbaffled tank) $\mu = 1$ cP.

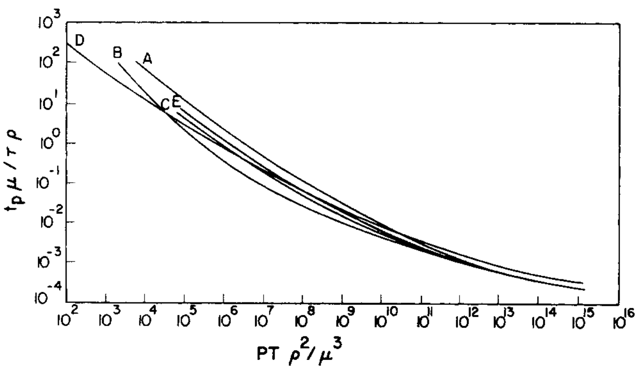

Fig. 8. Correlation of data according to Equation (5). (Legend for impeller-types same as Figures 6 and 7.)

nificant differences between the impellers were found. This conclusion is further supported by Figure 8 which summarizes all the data in terms of a correlation recently proposed by Zlokarnik (9):

$$\frac{Pt_p{}^3}{\rho T^5} = f\left(\frac{T^2\rho}{\mu t_p}\right) \qquad (5)$$

Mechanism of mixing by tubular agitators

In order to evaluate separately the external and internal pumping effects of the tubular impellers, the disc-type tubular impeller was made (of transparent plastic) so that the external pumping effect was minimized. The curvature of the channels in the disc were calculated using centrifugal pump theory. The performance of this impeller under viscous conditions was very poor as can be seen in Figure 5 where at a Reynolds number $Re = 2 \times 10^2$ there was an unfavorably sharp increase in Nt_p as Re decreases. In several experiments, dyestuff was injected in the center of the tank and its velocity of rise in the transparent plastic tube was measured. In one particular experiment at viscosity = 760 cP, the dyestuff passed through the tube at a rate of 22.3 cu.cm./min. Even assuming that for complete mixing the liquid in the vessel should circulate once (*8*) through the body of the agitator, complete mixing on this basis would be achieved in 5 hr. which is considerably longer than the measured times. Furthermore, when the tubular agitators were run with the arms plugged with stoppers no measurable decrease in mixing efficiencies were observed. Thus, it is concluded that internal pumping does not contribute significantly to the mixing process with these impeller types.

It would appear that these tubular agitators effectively acted as rod-type agitators whose efficiency was unexpected. Further experiments with specially made single rod-type agitators* (with the stirring rod mounted at the end of the shaft) have shown that the stirring efficiency-power consumption relationship is insensitive to the relative position of the stirring rod inside the tank. Visual observation of the flow pattern induced by this type of agitator showed a spiralling effect extending from the bottom of the container to the open surface of the liquid (Figure 9) which promoted efficient mixing in the tank.

Mixing of Non-Newtonian Liquids

The data obtained in this study were taken in the transition flow region for the pseudoplastic fluids. The power measurements were plotted using a generalized Reynolds number as before [Equation (3)]. The data for the conventional agitators (for example, see Figure 2) fitted the curves for Newtonian fluids as previously found (*6*). Figure 3 for the tubular agitators also shows similar agreements on this basis.

In contrast, the mixing-times in the pseudoplastic fluids were not the same but were higher than those for Newtonian fluids for laminar-flow mixing (see Figures 4 and 5). Since eddies decay more rapidly in the pseudoplastics, lower mixing is to be expected as Re decreases in these systems as found.

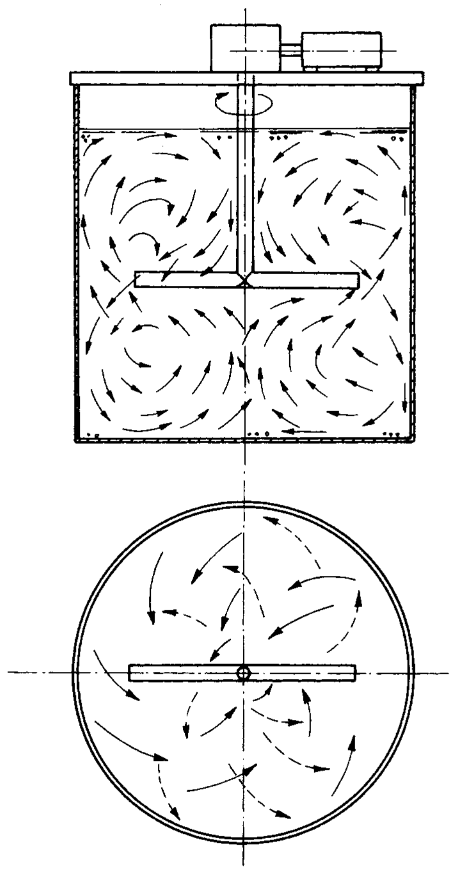

Fig. 9. Illustration of flow pattern within mixing vessel (single tubular or rod-type agitator).

* Patent applied for.

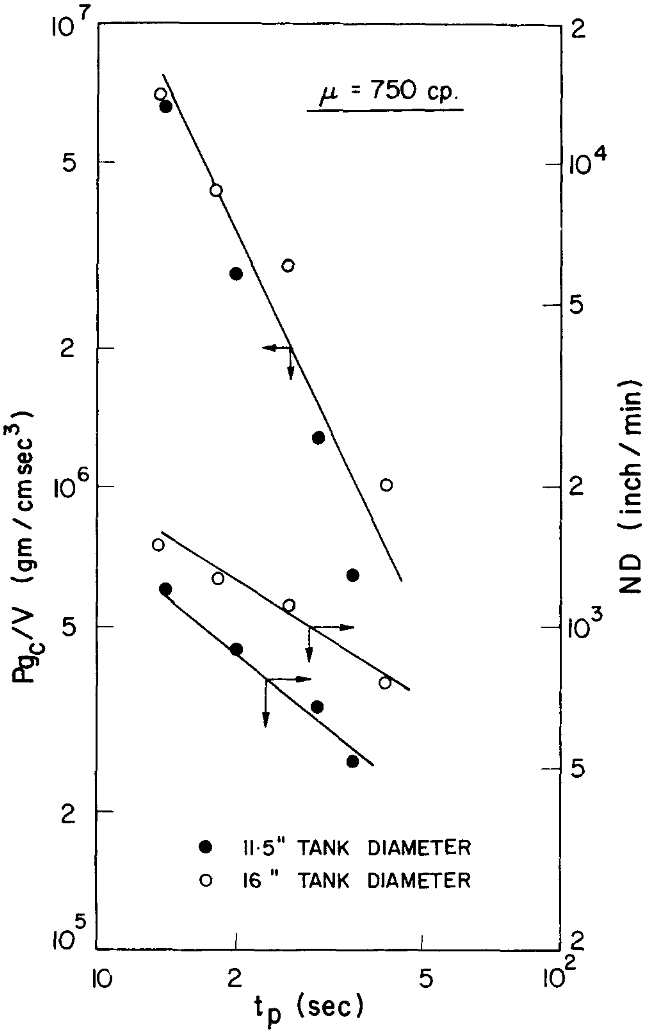

Fig. 10. Mixing time as a function of power per unit volume and impeller tip velocity. Tubular-agitator I (unbaffled tanks).

Scaling of Tubular Agitator

For practical purposes it is important to find a criterion for transferring the results of an experimental to a commercial scale. Mixing times were measured in one more geometrically similar vessel, 16 in. in diam., with the tubular agitator I. The results are reported in Figure 10 which shows that equal power consumption per unit volume and not tip speed of the impeller, as suggested by some workers, give equal mixing times. Furthermore, it is to be noted that the mixing time for this type of agitator varies as the square-root of the power input in contradistinction to the first-power dependence as found by Oldshue et al. (*10*) for the blending of liquids with propellers.

CONCLUSIONS

Examples taken from extensive experimental data of a recent thesis (*11*) reveal the following:

1. Power and mixing time for both the conventional and novel tubular agitators can be correlated in the usual manner by $Po = f(Re)$, $Nt_p = f'(Re)$.
2. Mixing under unbaffled conditions was found to be more efficient in the laminar and transient flow regions, while under turbulent conditions of flow, better mixing was achieved in the vessel provided with baffles.
3. Of all the stirrers tested the tubular agitator I was the most efficient one for the mixing of liquids between viscosities of 100 and 10,000 cP. For higher viscosities the helical agitator was more efficient.
4. The usual basis of power consumption per unit volume appears to be a convenient way of scaling-up of the tubular-agitator system for equal mixing times.
5. For pseudoplastic fluids, the power consumption can be unified for all the agitator types by correlations for Newtonian liquids by using a generalized Reynolds number. However, mixing times increase more rapidly with decreasing generalized Reynolds numbers with pseudoplastics than with Newtonian liquids and the data for both do not follow a single correlation.

ACKNOWLEDGMENT

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NOTATION

- B = width of baffle; non-Newtonian mixing factor defined in (*6*)
- C = clearance of agitator from bottom of vessel
- C' = clearance of arms from vessel bottom
- C_1 = clearance of upper arms (dual impellers) from vessel bottom
- C_2 = clearance of lower arms (dual impellers) from vessel bottom
- D = diameter of agitator
- g = acceleration due to gravity
- h = height of turbine or helical-ribbon blade
- K = fluid consistency coefficient defined by Equation (1)
- n = fluid flow behaviour index defined by Equation (1)
- N = rotational speed
- P = power consumption
- Po = power number defined by Equation (3)
- Re = generalized Reynolds number defined by Equation (3)
- S = cup-to-bob ratio in Couette viscometer
- t = mixing time in general
- t_p = mixing time in which $p\%$ of vessel volume is mixed
- T = diameter of mixing tank
- V = volume of tank contents
- w = width of turbine blade
- ρ = density
- $\dot{\gamma}$ = shear rate

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